Spatial-Energy Characteristics of Focused Modes of Metallic Terahertz Laser Resonator

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***Abstract* — Theoretically, based on the Rayleigh-Sommerfeld vector theory and using the radiation of a waveguide terahertz laser (λ = 432.6 μm), the physical characteristics of moderate and sharp focusing of radiation beams excited by laser cavity modes based on a circular metal waveguide for various spatial polarizations of the field were studied. It is shown that in the focal region of the lens, in the case of sharp focusing of the radiation beam excited by the radially polarized TM_{01} mode, a significant increase in its axial intensity is observed, due to a significant increase in the intensity of the longitudinal component of the field of this mode in this case. The obtained results expand the knowledge about the features of the focus of the laser beams of terahertz.**

Keywords — focusing; terahertz laser; modes; spatial polarization; field component

I. Introduction

A new direction in laser physics, which has received considerable development recently, is the creation of laser beams with inhomogeneous spatial polarization of radiation. Polarization of two-dimensional coherent light radiation at each point of space represents an invisible for the eye, but informative field "image." Vector beams with different spatial polarization field attract great interest recently because of their interesting physical characteristics and potential applications [1, 2]. The active interest of researchers to the beam with an inhomogeneous distribution of the axially symmetric polarization became apparent after a series of studies in which it was shown theoretically that such focused beams allow to improve the shape of the focal spot, to reduce the focal length and have a longer depth of focus area [3–6]. Modes with radial and azimuthal directions of polarization are of greatest practical interest [7]. In the optical range, radially polarized laser beams are used for metal cutting, particle acceleration, photonic crystal production, near-field microscopy, particle capture, surface plasmon polariton excitation. Laser beams with an azimuthal type of polarization of radiation are very useful for such applications as drilling, welding, excitation of ring metamaterial resonators, control of the motion of atoms. In the terahertz (THz) band, beams with inhomogeneous spatial polarization are proposed to be used to diagnose the surface of materials, thin films, biological objects, in information transmission and processing systems, communication systems, to achieve subwave resolution in tomography, etc.

Earlier in [8, 9], we studied focusing features of radiation beams excited by the modes of a THz laser cavity based on a circular hollow dielectric waveguide. The purpose of this paper is to study the structure features of the laser radiation beam field excited by the modes of a THz laser resonator based on a circular metal waveguide in the region of their focusing under different types of spatial polarization of these modes.

II. Theoretical Relationships

The propagation of laser radiation in free space along the 0z axis is described by the well-known Rayleigh-Sommerfeld integrals in the nonparaxial approximation [10, 11]. In the cylindrical coordinate system, they have the form:

$$\begin{aligned}
E_r(\rho,\theta,z) &= -\frac{ikz}{2\pi\xi^3}e^{ik\xi}\int_0^{\infty}\int_0^{2\pi}\begin{bmatrix}E_r(r,\phi+\theta,0)\cos\phi-\\ -E_\phi(r,\phi+\theta,0)\sin\phi\end{bmatrix}\times\\
&\qquad\times\exp\left(\frac{ikr^2}{2\xi}\right)\exp(-i\gamma r\cos\phi)\,rdrd\phi,\\
E_\phi(\rho,\theta,z) &= -\frac{ikz}{2\pi\xi^2}e^{ik\xi}\int_0^{\infty}\int_0^{2\pi}\begin{bmatrix}E_r(r,\phi+\theta,0)\sin\phi-\\ -E_\phi(r,\phi+\theta,0)\cos\phi\end{bmatrix}\times\\
&\qquad\times\exp\left(\frac{ikr^2}{2\xi}\right)\exp(-i\gamma r\cos\phi)\,rdrd\phi,\\
E_z(\rho,\theta,z) &= \frac{ik}{2\pi\xi^2}e^{ik\xi}\int_0^{\infty}\int_0^{2\pi}\begin{bmatrix}E_r(r,\phi+\theta,0)(r-\rho\cos\phi)+\\ +E_\phi(r,\phi+\theta,0)\rho\sin\phi\end{bmatrix}\times\\
&\qquad\times\exp\left(\frac{ikr^2}{2\xi}\right)\exp(-i\gamma r\cos\phi)\,rdrd\phi,
\end{aligned}\tag{1}$$

where $k = 2\pi/\lambda$ is wave number, λ is wavelength, ρ,θ,z – cylindrical coordinates in the observation plane, r and ϕ — polar coordinates in the area of the initial field, $\xi = \sqrt{z^2+\rho^2}$, $\gamma = k\rho/\xi$.

The modes of the investigated laser cavity coincide with the modes of a circular metal waveguide. Therefore, in the initial plane (at the exit mirror of the laser) we define the radiation in the form of symmetric azimuthally and radially polarized modes of a circular metallic waveguide of radius a. The normalized components of the electromagnetic fields of these modes in the plane of the source $z = 0$ have a known form [12]:

$$\begin{cases} \vec{E}_r^{TE_{mn}}(r,\phi) = \pm\vec{r}_0 A_{mn}\dfrac{m}{r}J_m\left(\chi'_{mn}\dfrac{r}{a}\right)\begin{Bmatrix}\sin m\phi\\ \cos m\phi\end{Bmatrix}, \\ \vec{E}_\phi^{TE_{mn}}(r,\phi) = \vec{\phi}_0 A_{mn}\dfrac{\chi'_{mn}}{a}J_m{}'\left(\chi'_{mn}\dfrac{r}{a}\right)\begin{Bmatrix}\cos m\phi\\ \sin m\phi\end{Bmatrix}, \end{cases}$$
$$\begin{cases} \vec{E}_r^{TM_{mn}}(r,\phi) = -\vec{r}_0 B_{mn}\dfrac{1}{a}J'_m\left(\chi_{mn}\dfrac{r}{a}\right)\begin{Bmatrix}\cos m\phi\\ \sin m\phi\end{Bmatrix}, \\ \vec{E}_\phi^{TM_{mn}}(r,\phi) = \vec{\phi}_0 B_{mn}\dfrac{m}{\chi'_{mn}r}J_m\left(\chi_{mn}\dfrac{r}{a}\right)\begin{Bmatrix}\sin m\phi\\ \cos m\phi\end{Bmatrix}, \end{cases} \tag{2}$$

where $\vec{r}_0, \vec{\phi}_0$ are the unit vectors of the cylindrical coordinates in the *r*- and ϕ-directions; *m* and *n* are integer azimuthal and radial wave indexes, respectively, χ'_{mn} is the *n*th root of the equation $J'_m(\chi'_{mn})=0$, χ_{mn} is the *n*th root of the equation $J_m(\chi_{mn})=0$, J_m is the Bessel function of the first kind of the *m*th order, A_{mn} and B_{mn} are the normalizing factors:

$$A_{mn} = \left[\frac{\varepsilon_m}{\pi(\chi_{mn}^2 - m^2)}\right]^{1/2}\frac{1}{J_m(\chi'_{mn})},$$

$$B_{mn} = \left[\frac{\varepsilon_m}{\pi}\right]^{1/2}\frac{1}{J_{m+1}(\chi_{mn})},$$

$$\varepsilon_m = \begin{cases}1 \text{ at } m = 0;\\ 2 \text{ at } m \neq 0.\end{cases}$$

The functions satisfy the following orthogonality conditions:

$$\int_0^a\int_0^{2\pi}\vec{E}_{mn}^{TE}(r,\phi)\vec{E}_{lp}^{TE}(r,\phi)rdrd\phi =$$
$$=\int_0^a\int_0^{2\pi}\vec{E}_{mn}^{TM}(r,\phi)\vec{E}_{lp}^{TM}(r,\phi)rdrd\phi =$$
$$=\begin{cases}1, m = l, n = p,\\ 0, m \neq l, n \neq p,\end{cases}$$
$$\int_0^a\int_0^{2\pi}\vec{E}_{mn}^{TE}(r,\phi)\vec{E}_{lp}^{TM}(r,\phi)\,rdrd\phi = 0$$

Let the radiation in the initial plane be given in the form of lower symmetric azimuthally and radially polarized TE_{01} и TM_{01} modes of a circular metallic waveguide. A lens of radius a_l is located at the output of this waveguide, which we describe using the phase correction function $Ph(r) = \exp\left(\dfrac{-i\pi r^2}{\lambda F}\right)$, where *F* is the focal length of the lens. Using the Rayleigh-Sommerfeld integral transforms (1) and the well-known integral

$$\int_0^{2\pi}\begin{Bmatrix}\cos(m\phi)\\ \sin(m\phi)\end{Bmatrix}\exp[-ix\cos(\phi-\theta)]d\phi = 2\pi(-i)^m J_m(x)\begin{Bmatrix}\cos(m\theta)\\ \sin(m\theta)\end{Bmatrix}$$

we obtain the field components of these modes in free space at a distance z from the lens.

The field components for the azimuthally polarized symmetric TE_{01} mode have the form:

$$\begin{aligned} &E_r(\rho,\theta,z) = 0,\\ &E_\phi(\rho,\theta,z) = -\frac{kz}{\xi^2}e^{ik\xi}A_{01}\frac{\chi_{01}}{a}\int_0^a J_1\left(\chi_{on}\frac{r}{a}\right)\times\\ &\qquad\times J_1(\gamma r)\exp\left(\frac{ikr^2}{2\xi}\right)Ph(r)rdr,\\ &E_z(\rho,\theta,z) = 0. \end{aligned} \tag{3}$$

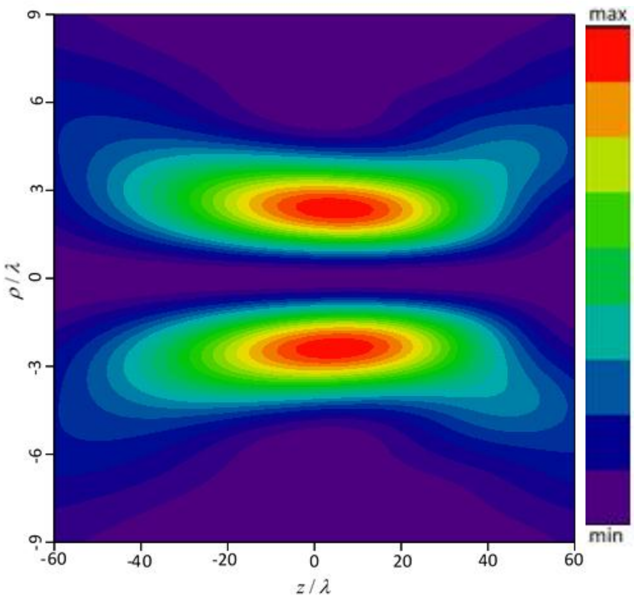
The field components for the radially polarized symmetric TM_{01} mode have the form:

$$\begin{aligned} &E_r(\rho,\theta,z) = \frac{kz}{\xi^2}e^{ik\xi}B_{01}\int_0^a J_1\left(\chi_{01}\frac{r}{a}\right)\times\\ &\qquad\times J_1(\gamma r)\exp\left(\frac{ikr^2}{2\xi}\right)Ph(r)rdr\\ &E_\phi(\rho,\theta,z) = 0\\ &E_z(\rho,\theta,z) = -\frac{ik}{\xi^2}e^{ik\xi}B_{01}\frac{1}{a}\int_0^a J_1\left(\chi_{01}\frac{r}{a}\right)\times\\ &\times\left[J_0(\gamma r)r - i\rho J_1(\gamma r)\right]\exp\left(\frac{ikr^2}{2\xi}\right)Ph(r)rdr. \end{aligned} \tag{4}$$

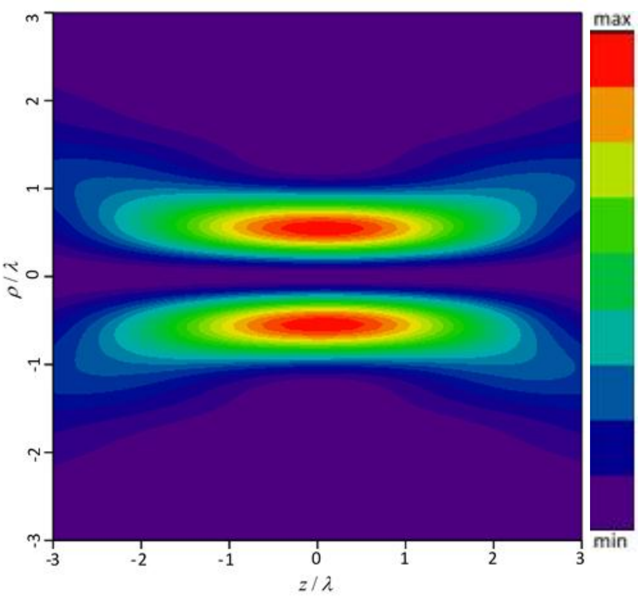
III. RESULTS AND THEIR DISCUSSION

Using the obtained expressions, the transverse distributions of the total intensity of the field $I=|E_x|^2+|E_y|^2+|E_z|^2$ and its individual components of the investigated resonator modes in the region of the minimum size of the focal spot of focused radiation beams were studied. The focal length of the lens F was chosen according to the conditions of moderate focusing (numerical aperture of the lens [13] NA=a_l/F = 0.2, a_l is the lens radius) and sharp focusing (NA = 0.7). The wavelength of the radiation was chosen in the middle part of the terahertz range λ = 432.6 μm (generation line for laser with optical pumping on the HCOOH molecule). The diameter of the waveguide is chosen to be 2a = 20 mm.

In Fig. 1 – 3 the field intensity distributions are given for modes with different spatial polarization of a THz laser resonator with a circular metal waveguide – symmetric azimuthally polarized TE_{01} mode and radially polarized TM_{01} modes at moderate and sharp focusing.



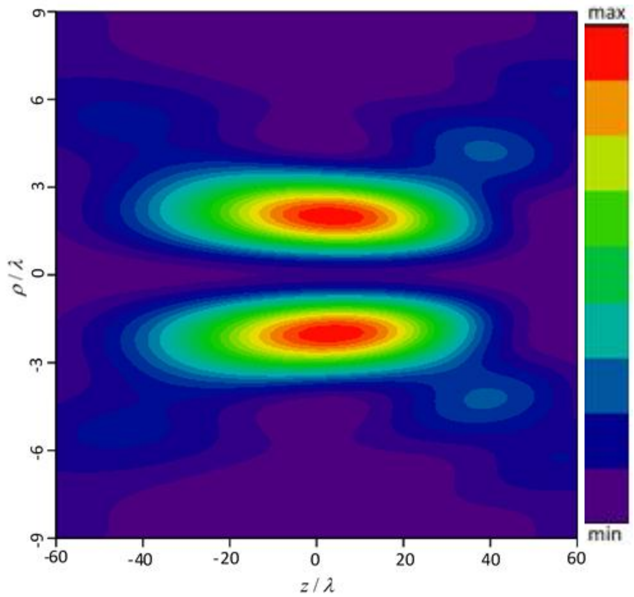
a)



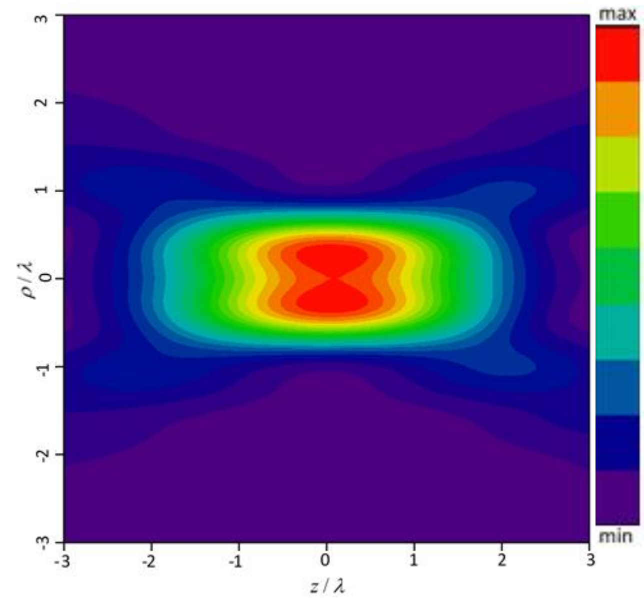
b)

Fig. 1. Calculated distributions of the total field intensity of the TE_{01} mode with moderate (a) and sharp (b) focusing.

Let's note some characteristic features. The transverse distribution of the total intensity of the field of the azimuthally polarized TE_{01} mode of the metal resonator in the region of the minimum size of the focused radiation beams retains an annular shape (Fig. 1) for both moderate and sharp focusing. These results coincide with the results obtained for the mode of a laser cavity based on a circular hollow dielectric waveguide



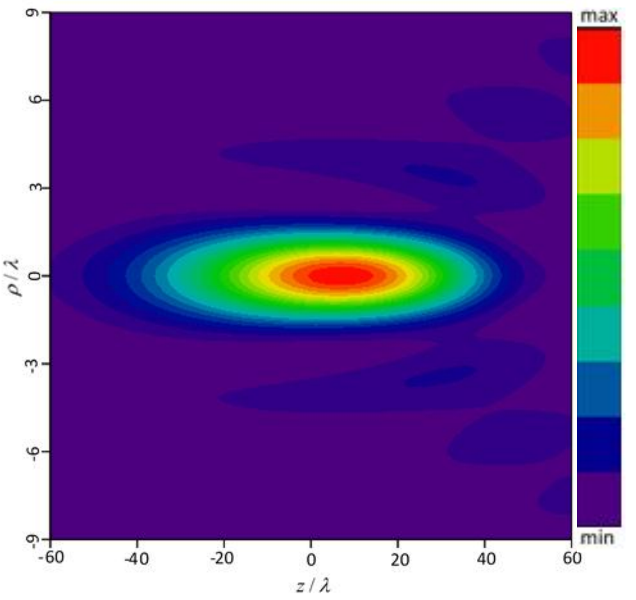
a)



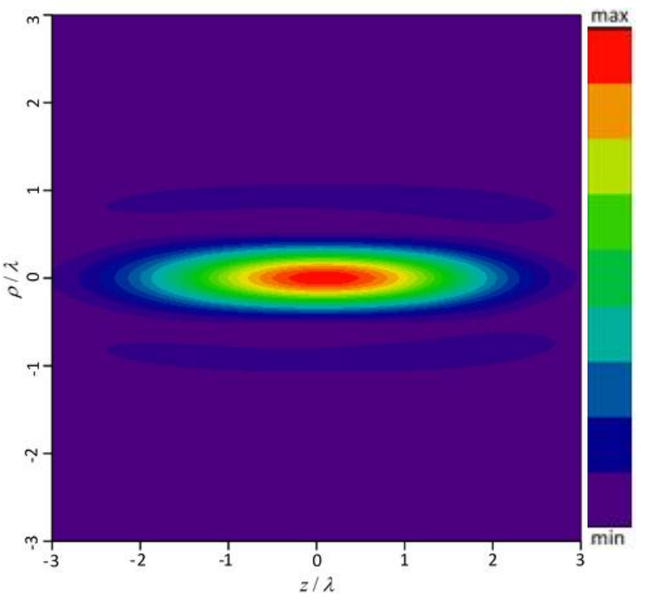
b)

Fig. 2. Calculated distributions of the total field intensity of the TM_{01} mode with moderate (a) and sharp (b) focusing.

For a radially polarized mode at sharp focusing in the transverse field distribution, a considerable increase in the axial intensity is observed (Fig. 2b), which is absent with moderate focusing (Fig. 2a). This is explained by the fact that the longitudinal component of the mode has a maximum of the field on the beam axis (Fig. 3).



a)



b)

Fig.3. Calculated distributions of the intensity of the longitudinal component of the field of the TM_{01} mode with moderate (a) and sharp (b) focusing.

To study the effect of the numerical aperture of a lens on the relative contribution of the longitudinal component of the field to the total mode intensity, calculations were carried out for different values of the NA parameter in the focal region of the lens in the expression:

$$\eta = \frac{\int_0^{2\pi}\int_0^{\infty} \left|E_z(\rho,\theta,z)\right|^2 \rho d\rho d\Theta}{\int_0^{2\pi}\int_0^{\infty}\left(\left|E_r(\rho,\theta,z)\right|^2 + \left|E_\phi(\rho,\theta,z)\right|^2 + \left|E_z(\rho,\theta,z)\right|^2\right)\rho d\rho d\Theta}$$

If NA = 0.2, its value is η = 1 %, then at NA = 0.7 it increases to 10 %.

CONCLUSIONS

The physical properties of moderate and sharp focusing in the free space for radiation beams excited by the modes of a THz laser cavity based on a circular metal waveguide with different spatial polarization of the field are theoretically investigated.

It is shown that in the focal region of the lens, in the case of sharp focusing of the radiation beam excited by the radially polarized TM_{01} mode, a significant increase in its axial intensity is observed, due to a significant increase in the intensity of the longitudinal component of the field of this mode in this case. For the azimuthally polarized TE_{01} mode, the focused beam of radiation retains an annular shape with both moderate and sharp focusing.

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